To compare our data on the maximum thickness of the two-phase layer in developed boiling with data on the limiting thickness of an individual bubble $\delta_{\ell m}$, we averaged values of δ_{tp} over q on sections of the relation $\delta_{tp}(q)$ and found the values of $\overline{\delta}_{tp}$ shown in Fig. 2. Values of $\delta_{\ell m}$ calculated by the method in [2] are plotted off the horizontal axis. It is evident that on the average the quantity $\overline{\delta}_{tp}$, changing in proportion to $\delta_{\ell m}$, somewhat exceeds the latter; as a first approximation, $\overline{\delta}_{tp} = \delta_{\ell m} + 0.2$ mm.

Thus, it can be concluded on the basis of the completed tests that in the developed nucleate boiling of liquids on a downward-turned horizontal surface, the maximum thickness of the two-phase layer may not be significantly greater than double the capillary constant and will be independent of the heat flux. In the case of sheet boiling, this conclusion is valid for the thickness of the vapor film which covers the heating surface.

LITERATURE CITED

1. N. Saki, S. Fukusako, and K. Torikosi, Teploperedacha, 100, No. 4, 67-72 (1978).
2. Yu. A. Kirichenko, K. V. Rusanov, and E. G. Tyurina, Inzh.-Fiz. Zh., 51, No. 5, 709-715 (1986).
3. M. M. Farahat and E. E. Madbouly, Int. J. Heat Mass Transfer, 20, No. 3, 269-277 (1977).
4. V. I. Tolubinskii, Heat Transfer during Boiling [in Russian], Kiev (1980).
5. Yu. A. Kirichenko, V. V. Tsybul'skii, M. L. Dolgoi, et al., Inzh.-Fiz. Zh., 28, No. 4, 581-588 (1975).

THEORETICAL STUDY OF THE STABILITY OF NUCLEATE BOILING AND PULSATIONS OF THE TEMPERATURE OF A WALL HEATED BY A HOT LIQUID

S. A. Kovalev and S. V. Usatikov

UDC 536.2

An analysis is made of the mechanism of development of temperature pulsations and conditions of the disturbance of the heat balance of a wall.

The heat-transfer crisis associated with the transition from nucleate boiling to sheet boiling poses a serious hazard, since it is accompanied by a sharp increase in wall temperature and may lead to burning of the heating surface. The boiling regime which exists under conditions of free convection is unambiguously determined by the difference between the temperature of the wall and the saturation temperature ($\Theta = T - T_s$). Proceeding on the basis of this fact, the authors of [1, 2] hypothesized that the notions of the stability of the temperature field of a heating element are equivalent. A sudden change in the temperature field in the event of a change of boiling regimes can be regarded as the result of disturbance of the heat balance of the wall [3]. Thus, to evaluate the stability of the temperature field, it is sufficient to solve the nonsteady heat-conduction problem for the wall. An analysis of the heat balance of the wall makes it possible to determine the region of stable operation and determine the magnitude of the deviations that are causing the shift in boiling regime.

Along with the heat-transfer crisis, danger is also presented by pulsation of the temperature of the wall. Such fluctuations may lead to fatigue failure of the wall material [4, 5]. As was shown in [4], analysis of the temperature field of a heating wall makes it possible to predict the conditions under which temperature fluctuations may be intensified.

The need for analysis of the temperature fields of walls has been noted in several other publications devoted to study of the stability of boiling regimes. For example, the

Institute of High Temperatures, Academy of Sciences of the USSR, Moscow.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 55, No. 5, pp. 803-810, November, 1988. Original article submitted June 11, 1987.

0022-0841/88/5505-1274$12.50

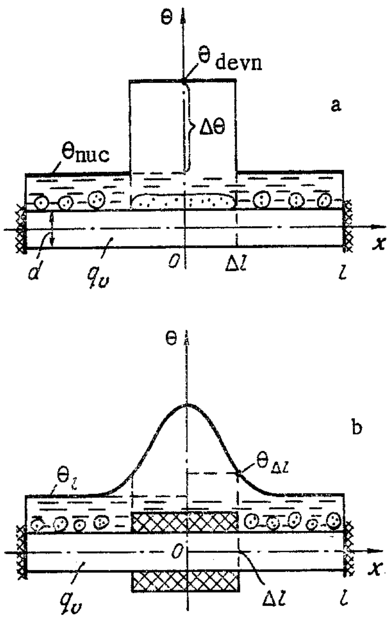


Fig. 1

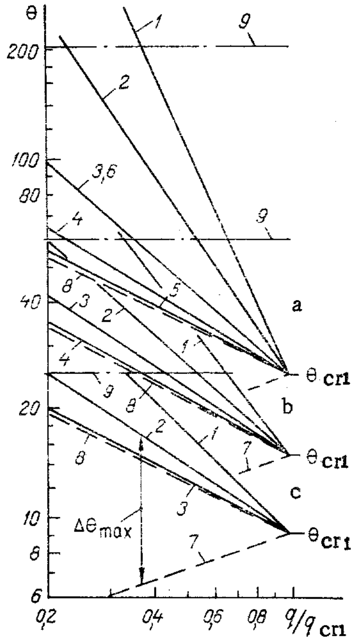


Fig. 2

Fig. 1. Temperature distribution along the rod at the initial moment, $\tau = 0$; $2\Delta\ell$ — region of sheet-boiling source.

Fig. 2. Dependence of the local critical overheating of the wall Θ_{devn} on pressure (a — p = 1 bar, b — 50, c — 100) and the dimensions of the hot spot: 1) $\Delta\ell$ = 1 mm; 2) 2; 3) 5; 4) 10; 5) more than 20 mm; 6) data from [2]; 7, 8) nucleate and translational boiling regimes; 9) limiting superheating of the liquid.

stability of boiling was evaluated with infinitesimal deviations of wall temperature in [6] and deviations of finite magnitude with the appearance of domains in [7]. In [8], the shift in boiling regimes was examined as a self-similar process.

Disturbance of the Heat Balance of the Wall. It is known that in the case of nucleate boiling on a massive, uniformly heated wall (slab, sphere) made of a material with a high thermal conductivity, the heat-transfer crisis is the result of disturbance of the hydrodynamic stability of the two-phase boundary layer at a heat flux equal to q_{cr1}. Here, all points of the heating surface have roughly the same temperature.

The mechanism of the crisis changes if the heating elements have relatively thin walls. As a result of deviations of the regime parameters or perturbations introduced from the outside during boiling, the local temperature of the wall may deviate from the mean steady-state value. A hot spot is formed on the wall. This spot is sometimes referred to as a sheet-boiling source. The hot spot either disappears (if its dimensions and the wall-temperature deviations are small) or grows (if its dimensions and the temperature deviations are substantial). In the latter case, the high-temperature front propagates along the surface until sheet boiling has embraced the entire surface. Let us calculate the temperature and dimensions of the hot spot at which the heat-transfer crisis occurs.

We will examine the process of boiling on the lateral surface of a horizontal rod of diameter d and length ℓ. For simplicity, we assume that temperature changes only along the rod. This assumption is valid if the condition $Bi = qd/(\lambda\Theta) < 1$ is satisfied. The temperature profile is described by the unidimensional equation

$$c\rho f \frac{\partial\Theta}{\partial\tau} = \lambda f \frac{\partial^2\Theta}{\partial x^2} - q_N u + qu \tag{1}$$

with the boundary and initial conditions (Fig. 1a)

$$\left.\frac{\partial\Theta}{\partial x}\right|_{x=0} = \left.\frac{\partial\Theta}{\partial x}\right|_{x=l} = 0, \tag{2}$$

TABLE 1. Parameters of the First and Second Crises in the Boiling of Water

p, bar	q_{cr1}, W/m²	q_{cr2}, W/m²	Θ_{cr1}, K	Θ_{cr2}, K
1	10^6	$4\cdot10^4$	25	125
60	$4\cdot10^6$	$14\cdot10^4$	15	75
100	$4\cdot10^6$	$14\cdot10^4$	9	45

at $\tau = 0$:

$$\Theta = \Theta_{nuc} + \Delta\Theta \quad \text{at} \quad 0 \leqslant x \leqslant \Delta l, \tag{3}$$

$$\Theta = \Theta_{nuc} \quad \text{at} \quad \Delta l < x \leqslant l, \tag{4}$$

where $q = q_v f/u$; q_v is the volumetric heat flux.

We isolate three regimes on the boiling curves $q_N(\Theta)$ and we adopt a different heat-transfer law for each regime:

Nucleate ($0 \leq \Theta/\Theta_{cr2} \leq 0.2$)

$$q_N/q_{cr1} = 125\,(\Theta/\Theta_{cr2})^3, \tag{5}$$

transitional ($0.2 < \Theta/\Theta_{cr2} \leq 1$)

$$q_N/q_{cr1} = 0.04\,(\Theta/\Theta_{cr2})^{-2},$$

sheet ($1 < \Theta/\Theta_{cr2}$)

$$q_N/q_{cr1} = 0.04\,(\Theta/\Theta_{cr2}).$$

The values of the constants are shown in Table 1.

As was shown in [9], the presence of the temperature gradient on the heating surface leads to some distortion of the boiling curve. For example, heat transfer is intensified in the region of the second boiling crisis. Equations (5) were constructed with allowance for the recommendations in [9].

Let a temporary disturbance of the heat-transfer regime on a certain section of a rod of the length $2\Delta\ell$ at a certain moment of time $\tau = 0$ result in a deviation of wall temperature from the steady-state value Θ_{nuc} by the amount $\Delta\Theta$ (Fig. 1a). This temperature profile is taken as the initial condition (3)-(4). We then calculate the critical value $\Delta\Theta_{max}$ for which the following conditions are satisfied: nculeate boiling is established on the rod at $\Delta\Theta < \Delta\Theta_{max}$, while the transition to sheet boiling occurs along the entire rod at $\Theta > \Delta\Theta_{max}$. The problem was solved numerically by the establishment method for a stainless steel rod with a diameter of 2 mm and length $\ell = 1$ m. Figure 2 shows the calculated critical values of the deviations $\Theta_{devn} = \Theta_{nuc} + \Delta\Theta_{max}$.

As might be expected, the quantity $\Delta\Theta_{max}$ depends on $\Delta\ell$. In the case of small $\Delta\ell$, $\Delta\Theta_{max}$ is large. The value of $\Delta\Theta_{max}$ initially decreases with an increase in $\Delta\ell$ but takes an asymptotic value at $\Delta\ell \to \ell$. According to [3, 10], the asymptotic value of the deviation corresponds to the temperature head in the transitional boiling regime (lines 8 in Fig. 2). The value of $\Delta\Theta_{max}$ decreases with an increase in q. Thus, for example, $\Delta\Theta_{max}$ decreases from 9 to 3°C with an increase in q from $0.4q_{cr1}$ to $0.8q_{cr1}$ (line 2 in Fig. 2c). At $q \to q_{cr1}$, any small increase in wall temperature may lead to the crisis, i.e., $\Delta\Theta_{max} \to 0$.

Kovalev [2] analytically determined the steady-state solutions of Eq. (1). These solutions are temperature profiles having fairly sharp maxima. It was shown that the steady-state solutions are unstable, the instability being manifest in the fact that such local deviations of temperature initiate a transition to sheet boiling over the entire rod. Critical values of the temperature maximum are shown in Fig. 2a on line 6. The laws discovered in [2] are in agreement with the results of our numerical calculations.

Several investigators [11-13] have suggested that a heat-transfer crisis will take place if the wall temperature in the region of the hot spot exceeds the temperature corresponding to the limiting superheating of the liquid. It is proposed that the crisis is thermodynamic in nature. However, calculations do not support this proposition. It can be seen from Fig. 2 that the coincidence of $\Delta\Theta_{max} + \Theta_{nuc}$ with $\Theta_{\ell s}$ is only a special case. Thus, for example, at $\Delta\ell = 1$ mm, they coincide only when $q = 0.4q_{cr1}$.

Consequently, for heat fluxes close to q_{cr1}, the quantity $\Delta\Theta_{max}$ will always be less than $\Theta_{\ell s} - \Theta_{nuc}$, and the hypothesis of the thermodynamic nature of the crisis will overestimate the safety threshold for overheating of the wall. The reliability of the above calculations can be checked against the following indirect empirical data. Let a steady nucleate regime be maintained on a heated horizontal rod. The surface of the rod is covered by insulation over a section of the length $2\Delta\ell$ of its central part, so this section is not in contact with the boiling liquid (the thermal insulation simulates a sheet-boiling source). Heat is removed from the insulated section by conduction. The temperature distribution along the rod is shown in Fig. 1b.

The stationary temperature distribution along the rod is described by Eq. (1) without the left side and by boundary condition (2). On the insulated section, $q_N = 0$, and we can write the following for the heat flux along the axis at the boundary of the insulated section:

$$\lambda \left. \frac{d\Theta}{dx} \right|_{x=\Delta l-0} = -q_v \Delta l = -4q \frac{\Delta l}{d}. \tag{6}$$

It is not hard to use Eq. (1) to also obtain an expression for the temperature gradient on the section cooled by the boiling liquid [2]. For the section adjacent to the insulated section, we have

$$\lambda \left. \frac{d\Theta}{dx} \right|_{x=\Delta l+0} = \sqrt{\frac{8\lambda}{d}} \sqrt{\int_{\Theta_l}^{\Theta_{\Delta l}} q_N \, d\Theta - q(\Theta_{\Delta l} - \Theta_l)}\,. \tag{7}$$

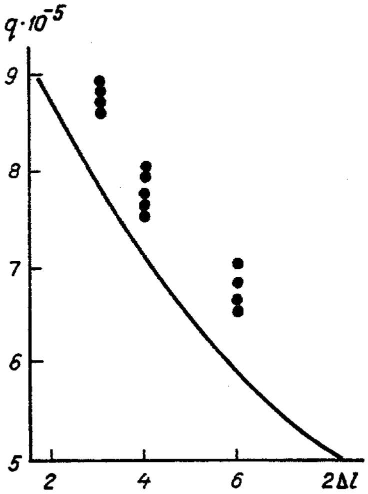
Here, Θ_ℓ and $\Theta_{\Delta\ell}$ are the temperature heads at the end of the rod and in the section $x = \Delta\ell$.

Let us compare (6) and (7). According to (6), the heat flow from the insulated section increases in proportion to $\Delta\ell$. The amount of heat that the boiling liquid can remove from the rod is limited by Eq. (7). If the heat flow from the insulated section exceeds this amount, then the heat balance will be disturbed. A hot spot is formed at the boundary of the insulated section, and the high-temperature section begins to move along the rod. It continues to do so until sheet boiling has spread over the entire surface. The heat-transfer crisis takes place. To determine the critical conditions for a prescribed value of q, we can use Eq. (7) to calculate the maximum value of $d\Theta/dx$ and use (6) to find $\Delta\ell$ for this value. The results of calculations performed for the case of the boiling of water at atmospheric pressure are shown in Fig. 3.

The tests involved study of the shift of boiling regimes on a horizontal rod with a diameter of 2 mm and a length of 300 m. The rod was made of stainless steel. The central section of the rod was thermally insulated over sections with lengths of 3, 4, and 6 mm. During the tests, we gradually increased the current passing through the rod (i.e., we increased the heat flux q) and we recorded the current at which the transition to sheet boiling took place. Figure 3 compares the experimental data with the calculation. The agreement is satisfactory.

Pulsations of Wall Temperature. If a hot liquid is used to heat a wall, then a heat-transfer crisis is avoided and the wall does not burn. However, appreciable fluctuations in wall temperature are seen in this case. The reasons for these pulsations are not yet entirely clear. Below we analyze a possible mechanism of reinforcement of small temperature pulsations in the heating liquid, the mechanism being linked to transitional boiling.

Let the liquid boil in a large volume (or in the presence of forced convection at low velocities and with a low relative enthalpy) on the outside surface of a tube through which a heating liquid (such as sodium) is being pumped. Relatively small pulsations of the temperature of the heating liquid take place. We want to see how the wall-temperature pulsations develop. For simplicity, we assume that the temperature changes only through the thickness of the wall.



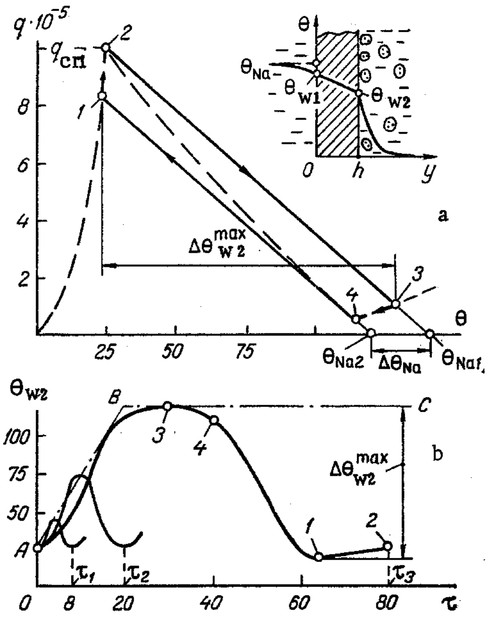


Fig. 3 Fig. 4

Fig. 3. Dependence of the critical heat flux on the length of the thermally insulated section; points show experimental data. Δℓ, mm.

Fig. 4. Cyclic thermal regime (1-2-3-4) of a wall heated by a hot liquid in the coordinates q–Θ (a) and over time (b); the dashed line shows the boiling curve.

Having assumed that the thickness of the wall h is much less than the radius of the tube, we write the heat conduction equation

$$\frac{\partial\Theta}{\partial\tau}=\frac{\lambda}{c\rho}\,\frac{\partial^2\Theta}{\partial y^2} \tag{8}$$

and the boundary conditions

$$-\lambda\frac{\partial\Theta}{\partial y}\bigg|_{y=0}=\alpha_{Na}\left[\Theta_{Na}-\Theta_{W1}\right], \tag{9}$$

$$-\lambda\frac{\partial\Theta}{\partial y}\bigg|_{y=h}=q_N\left(\Theta_{W2}\right). \tag{10}$$

Here, Θ_{W1} and Θ_{W2} are the temperatures of the inside and outside surfaces of the tube. The steady-state solution of problem (8)-(10) has the form

$$\Theta(x)=\Theta_{W2}+\frac{q_N\left(\Theta_{W2}\right)}{\lambda}\left[h-x\right],$$

where the quantity Θ_{W2} is determined from the heat-transfer equation

$$q_N\left(\Theta_{W2}\right)=\frac{\Theta_{Na}-\Theta_{W2}}{\dfrac{1}{\alpha_{Na}}+\dfrac{h}{\lambda}}. \tag{11}$$

A geometric interpretation of Eq. (11) is given in Fig. 4a. For the two prescribed values of sodium temperature Θ_{Na1} and Θ_{Na2}, Eq. (11) gives the lines 1-4 (Θ_{Na2}) and 2-3 (Θ_{Na1}). These straight lines will be referred to as the "heat conduction" lines of the wall. The slope of the lines is determined by the resistance of the tube wall and the coefficient of heat transfer from sodium to the wall. There may be from one to three solutions of problem (8)-(10), i.e., there may be from one to three points of intersection of the boiling curve

$q_N(\Theta)$ with a heat conduction line as Θ_{Na} changes from 1 to 3. It is not hard to show that the section of the boiling curves (line 2-4)

$$\frac{dq_N}{d\Theta} \leqslant \left(\frac{1}{\alpha_{Na}} + \frac{h}{\lambda}\right)^{-1}$$

is unstable. If the temperature of the heating liquid Θ_{Na} fluctuates and the range of fluctuation exceeds the interval $\Theta_{Na1}-\Theta_{Na2}$, then the boiling regimes will alternate on the heating surface in the closed cycle 1-2-3-4-1 (the path of the process is shown by arrows in Fig. 4a). The temperature of the heating surface of the tube will change within the range $\Delta\Theta_{w2}^{max}$, while the temperature of the heating liquid will vary within the range $\Delta\Theta_{Na}$. The ranges of fluctuation $\Delta\Theta_{w2}^{max}$ and $\Delta\Theta_{Na}$ do not coincide. If the boiling curve for the given liquid is slightly curved in the region of the transitional regime and is close to a heat-conduction line of the wall, then relatively small deviations in the temperature of the heating liquid will be accompanied by substantial fluctuations of the temperature heat $\Delta\Theta_{w2}$.

Let us solve problem (8)-(10). We will assume that the temperature of the heating liquid changes in accordance with the harmonic law

$$\Theta_{Na} = \Theta_{Na}^0 + \frac{\Delta\Theta_{Na}}{2} \sin\left(\frac{2\pi\tau}{\tau_0}\right),$$

and that the heat-transfer coefficients on the liquid-metal side are large $\Theta_{Na} = \Theta_{w1}$; the material of the tube is steel, while the wall thickness is 2 mm. Figure 4b shows results of calculations for $\Theta_{Na}{}^0 = 125°C$, $\Delta\Theta_{Na} = 2°C$ and three values of $\tau_0 = 8$, 20, and 80 sec. It is evident from the figure that wall temperature decreases from 95 to 20°C with a reduction in the period associated with the range of fluctuation.

The above relations are satisfactorily described by a formula obtained from the heat balance. In accordance with this formula, the maximum possible range of fluctuation $\Delta\Theta_{w2}^{max}$ is reached with the following period for the pulsations of the temperature of the heating liquid:

$$\tau_0^{max} \simeq \frac{h^2 c\rho}{\lambda}\left(1 + \frac{\lambda}{h\alpha_{Na}}\right)\frac{2\Delta\Theta_{w2}^{max}}{\Delta\Theta_{Na}}. \qquad (12)$$

If $\tau_0 > \tau_0^{max}$, then $\Delta\Theta_{w2} = \Delta\Theta_{w2}^{max}$. If $\tau_0 < \tau_0^{max}$, then $\Delta\Theta_{w2}$ is directly proportional to τ_0 and $\Delta\Theta_{Na}$.

Thus, numerical analysis of the temperature field of the wall made it possible to determine the mechanism of reinforcement of pulsations in the temperature of a wall heated by a hot liquid. The analysis also allowed us to establish quantitative relations. The final result is of practical interest, since steam generators designed to heat liquid metal are used in power engineering.

NOTATION

q, heat flux, W/m²; $q_N = q_N(\Theta)$, boiling curve; $2\Delta\ell$, dimension of the hot spot, m; u, f, perimeter and cross-sectional area of the rod, m, m²; h, d, ℓ, thickness of the plate, diameter of the rod, length, m; τ, time, sec; λ, thermal conductivity, W/(m·deg C); ρ, density, kg/m³; c, heat capacity, J/(kg·deg C); $\Theta = T - T_s$, temperature head, deg C; Θ_{nuc}, Θ_{cr1}, Θ_{cr2}, temperature head for nucleate boiling and the first and second boiling crises; $\Theta_{\ell s}$, limiting superheating of the liquid; Θ_{devn}, temperature head in the hot-spot region; Θ_{w2}, temperature head on the heating surface of the wall; $\Theta_{Na} = T_{Na} - T_s$, superheating of sodium; $\Delta\Theta_{Na}$, τ_0, amplitude and period of pulsations of sodium temperature; α_{Na}, coefficient of heat transfer from the heating liquid to the wall, W/(m²·deg C).

LITERATURE CITED

1. B. S. Petukhov and S. A. Kovalev, Teploenergetika, No. 5, 65-70 (1962).
2. S. A. Kovalev, Teplofiz. Vys. Temp., 2, No. 5, 780-788 (1964).
3. B. S. Petukhov, L. G. Genin, and S. A. Kovalev, Heat Transfer in Nuclear Power Plants [in Russian], Moscow (1986).
4. S. A. Kovalev and S. V. Usatnikov, Teplofiz. Vys. Temp., 23, No. 4, 771-780 (1985).

5. P. L. Kirillov, N. M. Turchin, N. S. Grachev, V. V. Khudasko, et al., At. Energ., 54, No. 5, 330-333 (1983).
6. K. Stephan, Chem. Ing. Tech., 38, No. 2, 112-117 (1966).
7. G. I. Abramov, S. I. Zakharchenko, and L. M. Fisher, Teplofiz. Vys. Temp., 24, No. 4, 736-742 (1986).
8. S. A. Zhukov, V. V. Barelko, and A. G. Merzhanov, Dokl. Akad. Nauk SSSR, 242, No. 5, 1064-1067 (1978).
9. G. M. Kazakov, "Study of heat transfer in the boiling of a liquid on a nonisothermal surface," Author's Abstract of Candidate's Dissertation, Engineering Sciences, Moscow (1971).
10. S. A. Kovalev, G. B. Rybchinskaya, and V. G. Vil'ke, Teplofiz. Vys. Temp., 11, No. 4, 805 (1973).
11. M. P. Friori and A. E. Bergles, Proc. 4th Int. Heat Transfer Conf., Vol. 6, B. 6.3, Paris (1970), p. 3.
12. V. A. Puzin," Construction of theoretical relations for the boiling crisis on the basis of experimental and theoretical studies of heat transfer in the forced motion of coolants," Author's Abstract of Candidate's Dissertation, Engineering Sciences, Moscow (1984).
13. S. B. Van Der Molen, F. W. B. M. Galjee, and P. A. Bozelie, Two-Phase Momentum and Mass Transfer in Chemical Process and Energy Engineering Systems, Vol. 2 (1979), pp. 825-832.